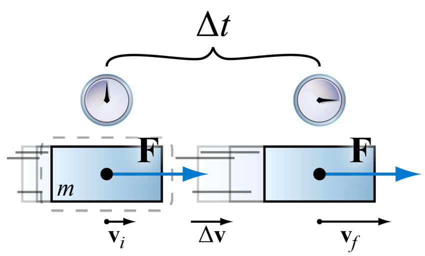
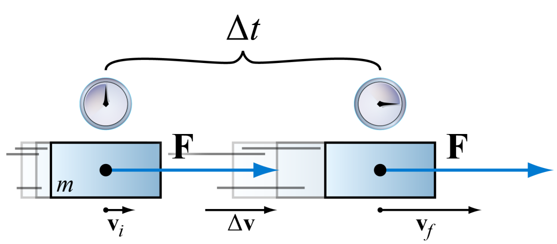
A net force can deliver an impulse that changes the $m\vec{\mathbf{v}}$ of an object

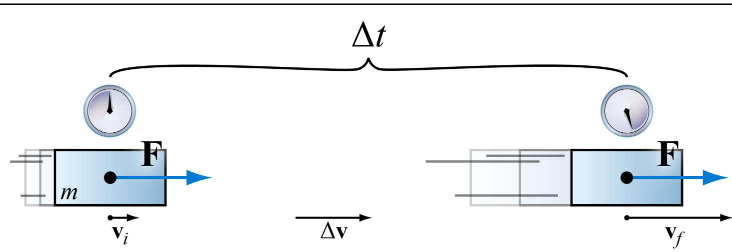
How much can I change the $\vec{\mathbf{v}}$ of an object of mass m by applying a constant force during some interval of time?



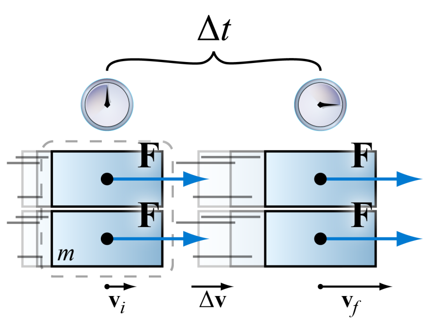
$$\Delta\vec{\mathbf{v}} \neq \vec{\mathbf{0}}$$



$$\uparrow |\vec{\mathbf{F}}| \Rightarrow \uparrow |\Delta\vec{\mathbf{v}}|$$



$$\uparrow \Delta t \Rightarrow \uparrow |\Delta\vec{\mathbf{v}}|$$



$$\uparrow |\textstyle\sum\vec{\mathbf{F}}| \Leftarrow \uparrow m$$

Deduced relationship

$$\begin{aligned}\left(\textstyle\sum\vec{\mathbf{F}}\right)\Delta t &= m\Delta\vec{\mathbf{v}} \\ &= m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right) \\ &= m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i \\ &= \Delta(m\vec{\mathbf{v}})\end{aligned}$$

Vocabulary

Momentum

$$\vec{\mathbf{p}} := m\vec{\mathbf{v}}$$

Impulse delivered by a force

$$\Delta\vec{\mathbf{J}}_{\mathrm{F}} := \vec{\mathbf{F}}_{\mathrm{AVG}}\Delta t$$

Impulse momentum-theorem

$$\vec{\mathbf{p}}_i + \overbrace{\sum_{\mathrm{F}} \Delta\vec{\mathbf{J}}_{\mathrm{F}}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}}}} = \vec{\mathbf{p}}_f$$

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Impulse delivered by a varying force

Consider the x-impulse delivered by an x-force of varying strength. Allow increments of time to be short enough so that, for each increment, the x-force is roughly constant.

$$\Delta J_{\mathrm{F},x,k} \approx F_{x,k}\Delta t$$

The total x-impulse delivered during a finite interval of time

$$\Delta J_{\mathrm{F},x} \approx \sum_k F_{x,k}\Delta t$$

is the signed area “under” the plot of F_x vs. t.

For AP Physics C,

$$\Delta J_{\mathrm{F},x} = \int_{t=t_i}^{t=t_f} F_x \, \mathrm{d}t$$